

HEAVY FLAVOURS LEPTONIC AND SEMI-LEPTONIC DECAYS ON THE LATTICE

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ABSTRACT

We present some results of the **ELC** (European Lattice Collaboration) study of leptonic and semi-leptonic decays of heavy-light mesons. Results on the decay constants f_D, f_{D_s} and f_B, f_{B_s} , the semi-leptonic form factors of $D \rightarrow K(K^*)$ and some preliminary estimates of form factors relevant for the $B \rightarrow \pi, \rho$ are summarized. The $1/M$ corrections to asymptotic scaling laws are discussed.

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1- Introduction

Lattice QCD, a non-perturbative method based on the QCD Lagrangien and its parameters, allows to predict many quantities such as hadron masses, the meson decay constants (DC), the B -parameter relevant for $B - \bar{B}$ mixing and the (SL) semi-leptonic form factors (FF) which play a crucial role in the determination of the C.K.M mixing matrix. It can be used also to test the scaling laws predicted by the HQET (Heavy Quark Effective Theory). This method is in continuous theoretical improvement (improved actions...) and since it is based on numerical simulations, there is actually an unlimited improvement in accuracy (dedicated machines...). But it has some limitation, for instance, we are limited to compute exclusive transitions with no more than one hadron in the final state. Moreover, with present computing machines, precise predictions can be made only in the quenched approximation and the systematic errors due to

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this latter cannot be evaluated. There are however some results obtained in the unquenched theory [1] suggesting that the systematic effect of quenching is small when we deal with heavy quarks. This is the study of weak decays of heavy mesons (composed by a heavy quark and a light one) into light ones.

2- Strategy to study heavy flavours on the lattice

The heavy quarks cannot move on a lattice because their masses are larger than the natural cut-off of the theory which is the inverse lattice spacing a^{-1} . To move on a lattice, a quark has to have a mass $m \ll a^{-1}$ (in practice, $m \leq 0.7a^{-1}$). The present lattice spacing ranges from 2.5 GeV to 4 GeV, and ELC's one is $a \simeq 3.6$ GeV. In the \overline{MS} scheme, $m_c \simeq 1.3$ GeV and $m_b \simeq 4.5$ GeV, so the study of the D meson is easy while it is presently impossible for a physical b quark to live on a lattice even the smallest lattice spacing is still larger than its Compton wavelength. So we are unable to study directly the B meson on the lattice. Nevertheless, indirect informations may be available following this strategy:

- We exploit the freedom in the choice of the masses (the Wilson hopping parameter κ -Wilson), then we create light mesons, the D meson and we increase the masses up to the limit $0.7a^{-1}$, we thus have fictitious “ D ” mesons, heavier than the D but still lighter than the B . We call this mass region the moving quark region.
- On the other hand, a method proposed by Eichten [2] allows one to put infinite mass on the lattice and the latter is considered in this approach as a static source of color.

Then one computes a physical quantity in the two mass regions (moving and static) and extracts it for the B meson by interpolation with the help of the scaling laws of the HQET. The value in the static limit reduces the uncertainties due to the extrapolation from the charm region. This method has shown to be very effective in the estimation of the pseudo-scalar DC. In the case of SL FF, at our large value of β (6.4), our small statistics doesn't allow to compute the FF in the static limit. Nevertheless, we can study their scaling behaviour and try an extrapolation to the b -quark. Our prediction concerning the B SL decays remain at a semi-quantitative level but it shows that the study is feasible.

Lattice Setup: this work has been done on a $24^3 \times 60$ lattice using the standard Wilson action at $\beta = 6.4$ for the gauge fields and the quark propagators[3], in the quenched approximation. Further details on the lattice calibration, fitting procedures, mass spectrum, extraction of matrix elements of local operators between the vacuum and meson states, e.g. $\langle M_P | \bar{Q} \gamma_5 q | 0 \rangle$, can be found in ref.[4].

3- Leptonic decays $D(B) \rightarrow \ell \nu$

The mass of any hadron can be obtained from the study of an appropriate euclidean correlation function as the coefficient of its exponential time dependence:

$$G(t) = \int d^3\mathbf{x} \langle \bar{u}(\mathbf{x}, t) \gamma_0 \gamma_5 c(\mathbf{x}, t) \bar{c}(\mathbf{0}, 0) \gamma_0 \gamma_5 u(\mathbf{0}, 0) \rangle \underset{t \rightarrow \infty}{\simeq} \frac{f_D^2 m_D}{2} e^{-m_D t}. \quad (1)$$

The determination of the expectation value in eq. 1 is a non perturbative problem which can be solved numerically. A second approach is based on the expansion of the heavy quark (H) propagator in inverse powers of the quark mass as proposed by Eichten [2]; the H is static and does not live effectively on the lattice but the quantity $f_H \sqrt{M_H}$ can be measured and is predicted to be independent of the heavy mass. The confrontation between the two methods is presented in Fig.1. When $M_H \rightarrow \infty$, the vector (V) and pseudoscalar (P) DC scale with the mass of the heavy quark, M_H , [2]-[5] ($M = M_P = M_V = M_H$, $\beta_0 = 11 - \frac{2}{3}N_f$):

$$\frac{M}{f_V} = f_P = \frac{C}{\sqrt{M}} \alpha_s(M)^{-2/\beta_0} \quad (2)$$

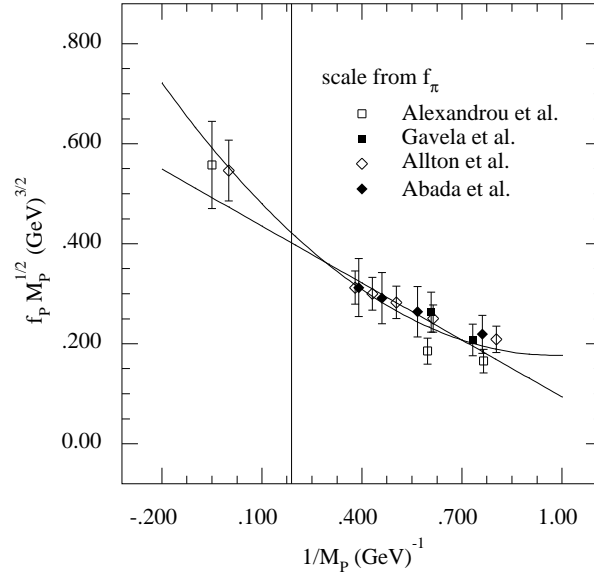


Fig.1. The $f_P \sqrt{M_P}$ is reported as a function of $1/M_P$. Results from other lattices are reported.

A horizontal line means eq. 2, a slope means corrections. The curves refer to a linear ($1/M$ correction) and a quadratic fit ($1/M^2$ correction). The vertical line identifies the physical B meson. The Figure.1 is very illustrative, the first point to notice is the consistency between the moving quark results and those from the static one. It appears also that there are large corrections to the asymptotic scaling behaviour (2). The main ELC results compared to the other lattice calculations and experiments are reported on Table 1 (2) concerning the D (B) meson.

Ref.	$f_D(\text{MeV})$	Ref.	$f_{D_s}(\text{MeV})$
ELC [4]	210 ± 15	ELC[4]	227 ± 15
APE (Clover)[6]	218 ± 9	(Clover)[6]	240 ± 9
Bernard et al.[7]	$208(9) \pm 35 \pm 12$	Bernard et al.[7]	$230(7) \pm 30 \pm 18$
UKQCD[8]	$185^{+4}_{-3} \text{ } ^{+42}_{-7}$	UKQCD[8]	212^{+4+46}_{-4-7}
-	-	EXP (WA75)[9]	$232 \pm 45 \pm 20 \pm 48$
-	-	EXP (CLEO2)[10]	$344 \pm 37 \pm 52 \pm 42$
-	-	EXP (ARGUS)[11]	267 ± 28

Table 1: Notice the agreement inside the lattice community. Taking the errors into account, there is a fair agreement with the experimental datas (“Clover” means continuum limit improved action).

Ref.	$f_B(\text{MeV})$	Ref.	$\frac{f_{B_s}}{f_{B_d}}$
ELC [4]	205 ± 40	ELC[4]	1.08 ± 0.06
APE (Static-Clover)[12]	$290 \pm 15 \pm 45$	(Static-Clover)[12]	$1.11(3)$
UKQCD (Static)[8]	$253^{+16}_{-15} \text{ } ^{+105}_{-14}$	(Static)[8]	1.14^{+4}_{-3}
Bernard et al.(Static)[7]	$235(20) \pm 21 \pm 12$	Bernard et al.(Static)[7]	1.11 ± 0.05
HEMCGC (Unquenched)[1]	200 ± 48		

Table 2: Notice that the HEMCGC unquenched result is not different from those using the quenched approximation (“Static” means infinite mass limit).

The B -parameter: the physical predictions for the $B - \bar{B}$ mixing depend on the value of the B -parameter of the heavy light $\Delta B = 2$ four quark operator [4]:

$$B_{D^0} = 1.05 \pm 0.08, \quad B_{B^0} = 1.16 \pm 0.07, \quad \frac{B_{D_s}}{B_{D_d}} \simeq \frac{B_{B_s}}{B_{B_d}} = 1.02 \pm 0.02 \quad (3)$$

Concerning the experimental implications, with the results quoted above, we can predict: $f_{B_d} \sqrt{B_{B_d}} = 220 \pm 40$ MeV which is the combination relevant for the CP violating effects in B decays and for $B - \bar{B}$ mixing amplitude.

4- Semi-leptonic decays $D(B) \rightarrow K, K^*(\pi, \rho)\ell\nu$

From the study of three-point correlation functions [13], one extracts the weak current matrix elements for a given momentum transfer q :

$$\langle K | J_\mu | D \rangle = \left(p_D + p_K - \frac{M_D^2 - M_K^2}{q^2} q \right)_\mu f_K^+(q^2) + \frac{M_D^2 - M_K^2}{q^2} q_\mu f_K^0(q^2) \quad (4)$$

$$\begin{aligned} \langle K_r^* | J_\mu | D \rangle &= e_r^\beta \left[\frac{2V(q^2)}{M_D + M_{K^*}} \epsilon_{\mu\gamma\delta\beta} p_D^\gamma p_{K^*}^\delta + i(M_D + M_{K^*}) A_1(q^2) g_{\mu\beta} \right. \\ &\quad \left. - i \frac{A_2(q^2)}{M_D + M_{K^*}} P_\mu q_\beta + i \frac{A(q^2)}{q^2} 2M_{K^*} q_\mu P_\beta \right], \end{aligned} \quad (5)$$

$q = p_D - p_{K(K^*)}$, $P = p_D + p_{K(K^*)}$ and e_r^β is the polarization vector of the K^* . $f_K^{+,0}$, V , $A_{1,2}$, A are dimensionless FF [14]. By varying the Lorentz component of the current (4), (5), the meson momenta and the K^* polarization, we can extract the 6 FF (only 4 are relevant in the decay rates). As said before, our study tries to extrapolate the FF to $B \rightarrow \pi, \rho$ decays using the scaling laws predicted by the HQET [15],[16]. On another hand, we have studied the q^2 dependance by putting a set of discret (volume is finite) momenta on the lattice for the final meson (the initial being at rest): $\vec{p}_k = (0, 0, 0)$ (for which $q^2 = q_{max}^2$), $(1, 0, 0)$, $(2, 0, 0)$, $(1, 1, 0)$ and $(1, 1, 1)$ (in lattice unit). We have computed the matrix elements (4, 5) and extracted the FF for different q^2 . All the computational details can be found in ref.[17]. At $\vec{p}_k = (1, 0, 0)$, we are near $q^2 = 0$ where we have the maximum of the partial widths of the D meson, but for the B , we are very far from $q^2 = 0$, then we had to perform an extrapolation to $q^2 = 0$, and the best that we could do was to use the nearest pole dominance[16]: $FF(q^2) = \frac{FF(0)}{1 - q^2/M_t^2}$, M_t being the mass of the meson exchanged in the t-channel.

D meson FF: the main ELC results compared to the other lattice calculations, theoretical models and experiments are reported on Table 3.

B meson FF: the HQET suggests that, when $M \rightarrow \infty$ at fixed \vec{p}_{K,K^*} and with $\vec{p}_{K,K^*} \ll M$, the FF scale as [15],[16]: $f^+ = M^{1/2} \gamma_+ \times \left(1 + \frac{\delta_+}{M}\right)$, $V = M^{1/2} \gamma_V \times \left(1 + \frac{\delta_V}{M}\right)$, $A_2 = M^{1/2} \gamma_2 \times \left(1 + \frac{\delta_2}{M}\right)$ and $A_1 = M^{-1/2} \gamma_1 \times \left(1 + \frac{\delta_1}{M}\right)$. The coefficients δ and γ are summarized in Table 4.

In Figure.2, we have plotted the FF as a function of $\frac{1}{M}$.

Ref	$f_K^+(0)$	$V(0)$	$A_1(0)$	$A_2(0)$
ELC.[17]	0.65 ± 0.18	0.95 ± 0.34	0.63 ± 0.14	0.45 ± 0.33
Lat.[13]	0.63 ± 0.08	0.86 ± 0.10	0.53 ± 0.03	0.19 ± 0.21
Lat.[18]	$.90 \pm .08 \pm .21$	$1.43 \pm .45 \pm .49$	$.83 \pm .14 \pm .28$	$.59 \pm .14 \pm .24$
QM.[19]	0.76	1.23	0.88	1.15
QM.[20]	$0.76 - 0.82$	1.1	0.8	0.8
SR.[21]	0.6 ± 0.10	—	—	—
SR.[22]	$0.6^{+0.15}_{-0.10}$	1.1 ± 0.25	0.5 ± 0.15	0.6 ± 0.15
Exp.[23]	$.70 \pm .08$	$.9 \pm .3 \pm .1$	$.46 \pm .05 \pm .05$	$0.0 \pm 0.2 \pm 0.1$

Table 3: $D \rightarrow K$ and K^* FF. “Lat.” refers to lattice QCD, “QM” to quark models, “SR” to QCD sum rules and “Exp.” to experiment.

\vec{p}	$\gamma_+ \text{ GeV}^{-1/2}$	$\gamma_V \text{ GeV}^{-1/2}$	$\gamma_1 \text{ GeV}^{+1/2}$	$\gamma_2 \text{ GeV}^{-1/2}$
(0, 0, 0)	—	—	0.96 ± 0.16	—
(1, 0, 0)	0.39 ± 0.25	0.29 ± 0.12	1.05 ± 0.25	0.44 ± 0.25
\vec{p}	$\delta_+ \text{ GeV}$	$\delta_V \text{ GeV}$	$\delta_1 \text{ GeV}$	$\delta_2 \text{ GeV}$
(0, 0, 0)	—	—	-0.33 ± 0.09	—
(1, 0, 0)	0.0 ± 1.1	1.9 ± 1.3	-0.46 ± 0.22	-0.6 ± 0.8

Table 4: The coefficients of the $1/M$ expansion of the FF.

5- Conclusion

After a first study of the leptonic heavy meson decays, we have looked at the semi-leptonic ones; we have done, for the first time on the lattice, the study of the scaling law predicted by the HQET concerning the FF: we get satisfactory quantitative results for the D meson. Concerning the B meson, we have good results near q_{max}^2 and the results we get at $q^2 = 0$ have very large errors, moreover, our predictions depend on the pole dominance approximation (more theoretical knowledge on q^2 behaviour of the FF is needed). But these results should be taken as a first indication of the feasibility of this method. Finally, $1/M$ corrections to the

scaling laws (leptonic and semi-leptonic decays) predicted by the HQET are large.

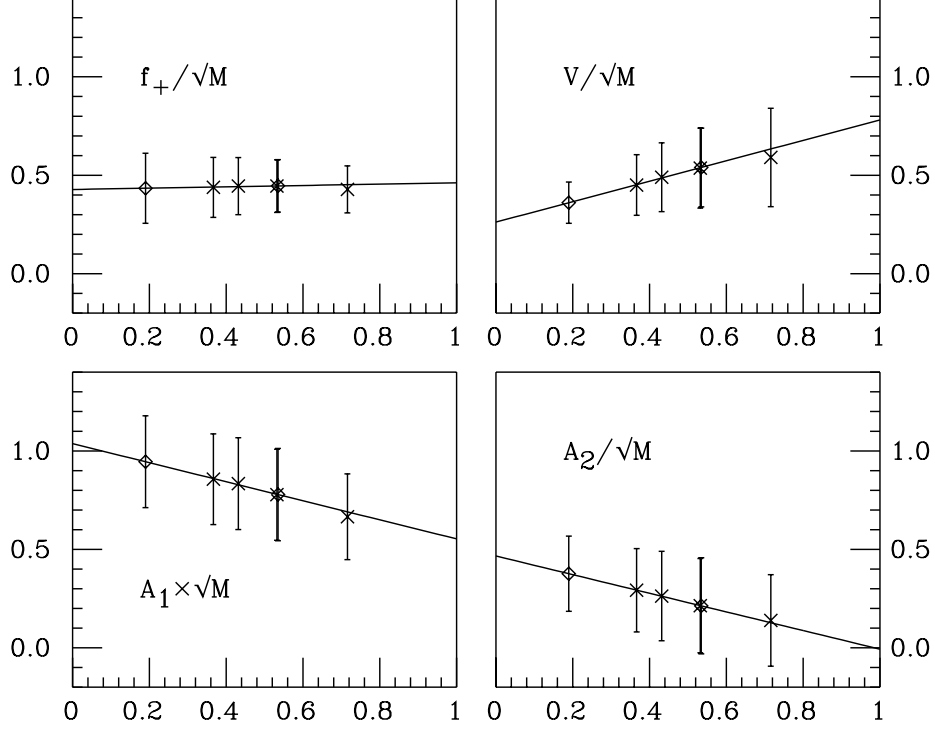


Fig.2. The crosses are the lattice points and the diamonds are the extrapolation to the D - and B -meson.

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